Introduction

The layout of VLSI chips is usually carried out in two steps: first, the building blocks are placed within the area of the chip, a step called placement, and then the blocks are connected. The problem of placement can be expressed as a graph partitioning problem. In this paper, we describe a heuristic algorithm which converts the graph partitioning problem into a two-dimensional space blocking problem. The objective is to minimize the total length of the connecting wires between the blocks.

The algorithm consists of two phases: the first is a rough partitioning of the blocks into a small number of clusters, and the second is a local optimization of the positions of the blocks within each cluster. The two phases are repeated until no further improvements can be made.

Acknowledgments

Received 15 September 1989
Received 15 February 1990

Shmulik Wiger

USA

Israel Karon

Department of Electrical and Computer Engineering, University of Massachusetts, Amherst, MA 01003.

Israel Cederbaum

Department of Electrical Engineering, Technion - Israel Institute of Technology, Haifa 33000, Israel.

Balanced Block Sizing for VLSI Layout

North-Holland
In Section 2, the space balancing problem is defined. In Section 3, we present the approach to find the one-dimensional layout, and the two-dimensional approach to find the one-dimensional layout. Section 4 consists of the solution for the one-dimensional problem. Section 5 proves the existence and optimality of the solution in Section 3, and Section 6 proves the existence and optimality of the solution in Section 5.
By \( w \), the width of the space along a path \( y \) in \( G \), is denoted by \( x(y) \). The real sum of all paths in a rectangle is denoted by \( \sum \). The area of the rectangle is denoted by \( \text{Area} \).

The horizontal adjacency graph \( H(G) \) of a graph \( G \) is defined as follows: Every vertex in \( H(G) \) is represented by a vertex in \( G \) whose width \( (x) \) is contained in the horizontal adjacency block. The vertical adjacency graph \( V(G) \) of a graph \( G \) is defined as follows: Every vertex in \( V(G) \) is represented by a vertex in \( G \) whose neighboring vertices are contained in the vertical adjacency block.
A horizontally balanced placement of the rectangles is called the one-dimensional space partitioning problem. Figure 3 illustrates a horizontally balanced placement of the rectangles. The rectangles are horizontally balanced if for every given horizontal placement, there exists an integer $q$ such that

$$q \geq \frac{n}{2} + \frac{1}{2}$$

and a rectangle is said to be horizontally balanced if

$$\min \{ f(r) : r \in R \} = \frac{q}{f}$$

We define $d^h = \min \{ d(r) : r \in R \}$ to be the horizontal distance of $R$. Vertical imbalance is defined similarly. The placement is said to be horizontally balanced if both horizontal and vertical imbalances are less than or equal to half the minimal horizontal and vertical distances of $R$, respectively. Let $d^v$ and $d^h$ denote the sets of arcs outgoing and incoming, respectively. By

Let $f^v$ and $f^h$ denote the sets of arcs outgoing and incoming, respectively.
As will be demonstrated by construction, for every initial placement there exists

3. Existence and uniqueness of one-dimensional space balancing

The adjacency relations is relaxed, the solution might be not unique as shown in

4. Horizontally balanced placement

Horizontal block spacing for AFLI layout
\[ |\sigma| \leq (\ell \mathcal{m} + (\ell \mathcal{m} + (\ell \mathcal{m} - (\ell \mathcal{m})) - (\ell \mathcal{m}))/w_0 \]
The average length of an arc along q in G is obtained from (8):

\[ \frac{|q_1| + |q_2| + |q_3|}{|q_1|d + |q_2|d + |q_3|d} = s \]

every arc along \( q \) in \( G \) is given by \( q_1, \ldots, q_n \) and \( q_1, \ldots, q_n \) respectively. Then, the length of the average length of the vertices of \( q \) and \( q \), respectively. Let \( d, d, \ldots, d \) and \( d, d, \ldots, d \) be the positions between the vertices \( q \) and \( q \), respectively.\)

The proof proceeds inductively on the order of the average length. First, the path to the minimal path whose average length is smaller than \( s \). This will contradict the selection of \( G \) as the path whose average length is larger than \( s \).

Proof. The proof proceeds inductively on the order of the average length. Let \( G \) be the path to the minimal path whose average length is larger than \( s \).

Lem 3.1. The length assigned to the arcs of the new adjacency graph is non-

Lemma 3.1. The length assigned to the arcs of the new adjacency graph is non-

The area of \( G \) is provided there in Lemma 3.2. Since the boundary of the length assigned to

2. The above procedure may yield multiple arc lengths, which in turn will result

in (8) is shown on a non-negative vertex. The disjointment of arc lengths is

3.2. Feasibility of the new adjacency graph

and the sixth (and final) iteration with the path \( p_1 \), 3. The fifth iteration with the path \( p_n \), 4. The fourth iteration with the path \( p_n \), 5. The third iteration with the path \( p_n \), 6. The second iteration with the path \( p_n \), 7. The first iteration with the path \( p_1 \) \( p_2 \), 8. The second iteration with the path \( p_2 \), 9. The first iteration with the path \( p_n \), 10. The second iteration with the path \( p_n \), 11. The third iteration with the path \( p_n \), 12. The fourth iteration with the path \( p_n \), 13. The fifth iteration with the path \( p_n \), 14. The sixth iteration with the path \( p_n \), 15. The seventh iteration with the path \( p_n \), 16. The eighth iteration with the path \( p_n \), 17. The ninth iteration with the path \( p_n \), 18. The tenth iteration with the path \( p_n \), 19. The eleventh iteration with the path \( p_n \), 20. The twelfth iteration with the path \( p_n \). The first example given in Fig. 1, the first time iterations in the bottom row, and the graph have the edges between any two vertices in the bottom row. In the edges between any two vertices in the bottom row. Hence, the graph have the edges between any two vertices in the bottom row.
are combinations of the above three and similar arguments lead to contradiction.

For which the ratio in (1) is smaller, the remaining six possible
and were already marked. Therefore, there was another undisturbed path between
vertices of (1), (2), and two paths from $\ell$ to $\ell$ and that have one path from
part from $\ell$ to $\ell$ and that have some
more such a situation is impossible. A chain possibility is that $\ell$, i.e. 

as shown in Fig. 6(a). Arguments similar to those used for the first induction step

so-called, which is a contradiction. A second possibility is that $\ell$, and that have no

not be on any path from $\ell$ to $\ell$ as shown in Fig. 6(b). Then, let us consider each one of them. Assume first that $\ell$ and $\ell$ do

and there are no possible paths for the relation between $\ell$ and $\ell$ where $\ell$ and $\ell$ are the end vertices of $\ell$. Let $\ell$ and $\ell$ do the same as above, which the above are

sum of all the paths from $\ell$ to $\ell$ as in (9) to (6). This yields (7) which contradicts the assumption of

\[
\frac{\left|F_{\ell}\right| + \left|G_{\ell}\right| + \left|H_{\ell}\right|}{(\ell')m + (\ell')m + (\ell')m - (\ell')m - (\ell')m - (\ell')m} = s
\]

is.

The vertices $\ell$ and $\ell$ are above the path in the context of $\ell$ and $\ell$.

\[
\frac{\left|F_{\ell}\right| + \left|G_{\ell}\right| + \left|H_{\ell}\right|}{\left|F_{\ell}\right| + \left|G_{\ell}\right| + \left|H_{\ell}\right|}
\]

\[
(\ell')m + (\ell')m + (\ell')m - (\ell')m - (\ell')m - (\ell')m < \left|F_{\ell}\right| + \left|G_{\ell}\right| + \left|H_{\ell}\right|
\]

then after some algebraic operations

From the contradictory assumption that $s > \ell$, we get

\[
\frac{\left|F_{\ell}\right|}{(\ell')m + (\ell')m + (\ell')m - (\ell')m - (\ell')m - (\ell')m} = \varepsilon s
\]

at a point of Lemma 3.1. The first induction step
We conclude with the following theorem.

**Theorem 3.3 (existence)** Given an initial placement of rectangles, its placement can always be preserved and be horizontally balanced.

We prove that \( G \) is a new horizontally adjacent graph isomorphic to the original \( G \), and hence \( G \) is balanced. From the construction procedure in Section 3.1 and from Lemma 3.1, we conclude balanced block structure for VLSI layout.
The supercritic ips and are used to distinguish between spaces (and similarly)

\[ (f)_D^s = s < d = (f)_D^s, \quad \forall \sigma \in \mathcal{D}, s, d \leq (\sigma)_D^s \]

whose lengths in \( D \) are greater than \( s \), namely.

**Theorem 3.4 (Uniqueness).** The horizontally balanced placement of a given initial

Assume first that the arc lengths above are different from those along \( D \).

Assume the other arc lengths of the supercritic points are identical in \( D \) and equal to

\[ \frac{p}{p+q} \]  

Recall that the lengths of all the paths from \( p \) to \( p+q \) are equal to \( p \) and that by

Similarly for the paths that are more than \( p \) and less than \( p+q \).

\[ 0 \leq t \leq 1 - \frac{p}{p+q} \]

In the same order the arc lengths obtained by the construction form two different sets of \( \frac{p}{p+q} \). Consider the path of \( D \) and their corresponding arc lengths.

**Proof.** Assume the contrary that the balancing is not unique. Let \( D \) and \( D' \) be

**Theorem 3.3 (Uniqueness).** The horizontally balanced placement of a given initial

The contrivance is unique.

Consider the paths of \( D \) and \( D' \). We may prove by induction on the order of \( \sigma \) that the paths of \( D \) and \( D' \) are identical in \( D \) and equal to

\[ \frac{p}{p+q} \]

Recall that the lengths of all the paths from \( p \) to \( p+q \) are equal to \( p \) and that by

Similarly for the paths that are more than \( p \) and less than \( p+q \).

\[ 0 \leq t \leq 1 - \frac{p}{p+q} \]

In the same order the arc lengths obtained by the construction form two different sets of \( \frac{p}{p+q} \). Consider the path of \( D \) and their corresponding arc lengths.

**Proof.** Assume the contrary that the balancing is not unique. Let \( D \) and \( D' \) be

**Theorem 3.3 (Uniqueness).** The horizontally balanced placement of a given initial

The contrivance is unique.

Consider the paths of \( D \) and \( D' \). We may prove by induction on the order of \( \sigma \) that the paths of \( D \) and \( D' \) are identical in \( D \) and equal to

\[ \frac{p}{p+q} \]

Recall that the lengths of all the paths from \( p \) to \( p+q \) are equal to \( p \) and that by

Similarly for the paths that are more than \( p \) and less than \( p+q \).

\[ 0 \leq t \leq 1 - \frac{p}{p+q} \]

In the same order the arc lengths obtained by the construction form two different sets of \( \frac{p}{p+q} \). Consider the path of \( D \) and their corresponding arc lengths.

**Proof.** Assume the contrary that the balancing is not unique. Let \( D \) and \( D' \) be

**Theorem 3.3 (Uniqueness).** The horizontally balanced placement of a given initial

The contrivance is unique.

Consider the paths of \( D \) and \( D' \). We may prove by induction on the order of \( \sigma \) that the paths of \( D \) and \( D' \) are identical in \( D \) and equal to

\[ \frac{p}{p+q} \]

Recall that the lengths of all the paths from \( p \) to \( p+q \) are equal to \( p \) and that by

Similarly for the paths that are more than \( p \) and less than \( p+q \).

\[ 0 \leq t \leq 1 - \frac{p}{p+q} \]

In the same order the arc lengths obtained by the construction form two different sets of \( \frac{p}{p+q} \). Consider the path of \( D \) and their corresponding arc lengths.

**Proof.** Assume the contrary that the balancing is not unique. Let \( D \) and \( D' \) be

**Theorem 3.3 (Uniqueness).** The horizontally balanced placement of a given initial

The contrivance is unique.

Consider the paths of \( D \) and \( D' \). We may prove by induction on the order of \( \sigma \) that the paths of \( D \) and \( D' \) are identical in \( D \) and equal to

\[ \frac{p}{p+q} \]

Recall that the lengths of all the paths from \( p \) to \( p+q \) are equal to \( p \) and that by

Similarly for the paths that are more than \( p \) and less than \( p+q \).

\[ 0 \leq t \leq 1 - \frac{p}{p+q} \]

In the same order the arc lengths obtained by the construction form two different sets of \( \frac{p}{p+q} \). Consider the path of \( D \) and their corresponding arc lengths.

**Proof.** Assume the contrary that the balancing is not unique. Let \( D \) and \( D' \) be

**Theorem 3.3 (Uniqueness).** The horizontally balanced placement of a given initial

The contrivance is unique.
Fig. 7. Two dimensional VLSI placement with immovable rectangles.
\[(\mu,\lambda)\alpha = (\mu,\lambda)^\circ + (\lambda,\mu)^\circ + (\lambda,\mu)^\circ + (\mu,\lambda)^\circ\]

(11)

Let us calculate the length of \( \lambda \) by combining (1) and

\[\alpha(\lambda)\gamma = (\mu,\lambda)^\circ \quad \gamma(\lambda)\gamma = (\mu,\lambda)^\circ\]

Induction hypothesis: there is.

\[\lambda \subseteq (\mu,\lambda)^\circ \]

According to the

In the graph corresponding to \( \lambda \), there exist two possibilities:

Assume now that \( \lambda \subseteq (\mu,\lambda)^\circ \) and there is not a single loop \( \lambda \).

From source to sink in the graph corresponding to the initial predecessor,

Therefore the vertices \( \lambda \) are the shortest among all the elements of the

\[\lambda \subseteq (\mu,\lambda)^\circ \]

And since the elements \( \lambda \) of the graph corresponding to the initial predecessor,

\[\lambda \subseteq (\mu,\lambda)^\circ \]

What is impossible.

\[\alpha(\lambda)\gamma = (\mu,\lambda)^\circ \quad \gamma(\lambda)\gamma = (\mu,\lambda)^\circ\]

Then from (1) and (2), we get that the length of \( \lambda \) is not greater than \( \lambda \).

Hence the graph corresponding to the initial predecessor,

\[\lambda \subseteq (\mu,\lambda)^\circ \]

Let us calculate the length of \( \lambda \) by the

\[\alpha(\lambda)\gamma = (\mu,\lambda)^\circ \quad \gamma(\lambda)\gamma = (\mu,\lambda)^\circ\]
4. A three-dimensional space balancing

Given a rectangle, let $d$ be the distance horizontally and $r$ be the distance vertically.

(1) $d + r = 2n \Rightarrow 1$.

(2) $\langle d, r \rangle = (n, n) \Rightarrow (n, n) \neq (n, n)$. 

(3) $\langle d, r \rangle = (n, 0) \Rightarrow (n, 0) \neq (0, n)$.

(4) $\langle d, r \rangle = (0, n) \Rightarrow (0, n) \neq (n, 0)$.

(5) $\langle d, r \rangle = (n, n) \Rightarrow (n, n) \neq (n, n)$.

(6) $\langle d, r \rangle = (n, 0) \Rightarrow (n, 0) \neq (0, n)$.

(7) $\langle d, r \rangle = (0, n) \Rightarrow (0, n) \neq (n, 0)$.

(8) $\langle d, r \rangle = (n, n) \Rightarrow (n, n) \neq (n, n)$.

(9) $\langle d, r \rangle = (n, 0) \Rightarrow (n, 0) \neq (0, n)$.

(10) $\langle d, r \rangle = (0, n) \Rightarrow (0, n) \neq (n, 0)$.

(11) $\langle d, r \rangle = (n, n) \Rightarrow (n, n) \neq (n, n)$.

(12) $\langle d, r \rangle = (n, 0) \Rightarrow (n, 0) \neq (0, n)$.

(13) $\langle d, r \rangle = (0, n) \Rightarrow (0, n) \neq (n, 0)$.

(14) $\langle d, r \rangle = (n, n) \Rightarrow (n, n) \neq (n, n)$.

(15) $\langle d, r \rangle = (n, 0) \Rightarrow (n, 0) \neq (0, n)$.

(16) $\langle d, r \rangle = (0, n) \Rightarrow (0, n) \neq (n, 0)$.

(17) $\langle d, r \rangle = (n, n) \Rightarrow (n, n) \neq (n, n)$.

(18) $\langle d, r \rangle = (n, 0) \Rightarrow (n, 0) \neq (0, n)$.

(19) $\langle d, r \rangle = (0, n) \Rightarrow (0, n) \neq (n, 0)$.

(20) $\langle d, r \rangle = (n, n) \Rightarrow (n, n) \neq (n, n)$.

(21) $\langle d, r \rangle = (n, 0) \Rightarrow (n, 0) \neq (0, n)$.

(22) $\langle d, r \rangle = (0, n) \Rightarrow (0, n) \neq (n, 0)$.

(23) $\langle d, r \rangle = (n, n) \Rightarrow (n, n) \neq (n, n)$.

(24) $\langle d, r \rangle = (n, 0) \Rightarrow (n, 0) \neq (0, n)$.

(25) $\langle d, r \rangle = (0, n) \Rightarrow (0, n) \neq (n, 0)$.
\[ (\omega(n) - 1) \mu^n = (\omega(n) + \cdots + \omega(n)) \mu^n \]

Theorem 4.1. The series of phenomena arising from the iterative application of balancing cycle constructions to the balanced placement.

1. Consideration of all balancing cycles constructed in the balanced placement.
This paper addressed the block spacing problem whose objective is to provide

**Conclusions and Further Research**

![Diagram of block spacing](image)

**Figure 8. An example illustrates the convergence of the balancing cycle.

- Initial Placement
- End of 1st cycle
- End of 2nd cycle
- End of 3rd cycle
- End of 4th cycle
- Final Placement

Verification could be obtained of the order would be reversed.

Cycle proceeds in the order of the rectangles' indices: Notice that a latter con-

- In general, convergence is guaranteed for an arbitrary balancing sequence, as long
- The series of adjacency graphs resulting from the balancing cycles

Corollary 4.2. The series of adjacency graphs resulting from the balancing cycles

A direct consequence from the proof of Theorem 4.1 is:
References

helpful comments and suggestions. The authors would like also to thank an anonymous reviewer for his
discussions with E. Sold from IBM Israel Scientific Center for

Acknowledgement

We have already seen the two dimensional space balancing problem may have

areas are length is minimized.

been and an algorithm for finding the path between two vertices along which the

while and efficient (polynomial) combinatorial solution to the space balancing prob-

problem is in practice, the second solution is an efficient heuristic algorithm

were discussed. One is a product of the experience gained of the experience gained

and in wireless applications which converges rapidly to the solution was presented.
The two algorithms for the solution of the problem was provided. The ex-


2] T. Ohtsuki, "A new scheme for circuit simulation using CDL, Dc, Layout Design and


E. Kuhl and A. Meret-Steinweizer, "Computer-aided layout in IC," Design and Verif-

ications.