The optimal controller for a system with stochastic inputs is designed to minimize the expected cost function. The system under consideration is given by the following differential equation:

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t) + C(t)w(t) \]

where \( x(t) \) is the state vector, \( u(t) \) is the control input, \( w(t) \) is the stochastic input, \( A(t) \), \( B(t) \), and \( C(t) \) are time-varying matrices.

The cost function to be minimized is given by:

\[ J = \mathbb{E} \left[ \int_0^T (x^T(t)Q(t)x(t) + u^2(t)R(t)) dt \right] \]

where \( Q(t) \) and \( R(t) \) are positive definite matrices.

The solution to the optimal control problem is obtained by solving the Hamilton-Jacobi equation:

\[ \frac{\partial J}{\partial x} + A^T(t)Q(t)x + B^T(t)R(t)u = 0 \]

The optimal control law is then given by:

\[ u(t) = -R(t)^{-1}B(t)^T \frac{\partial J}{\partial x} \]

This approach provides a systematic method for designing controllers that minimize the expected cost function.
The National Conversion of Electrical & Electronic Engineers in Israel

\[ (1) \quad x(t) = \phi(t) u(t) \]

\[ (2) \quad z(t) = N(t)x(t) + k(t) \]

Equations (1), (5) and (6) are combined to an augmented system equation:

RESULTS

\[ \text{This model has two main characteristics:} \]

\[ \text{First - the filter part and the gain part of the control controller,} \]

\[ \text{Second - we can easily design the controller.} \]

\[ \text{Assume the following form of the controller:} \]

\[ \text{where } \mathbf{z}(t) \text{ is the least-square estimate of } x(t) \]

\[ \text{This problem has also a well-known solution using Kalman optimal filter:} \]

\[ \{ \phi(t) \theta(t) \}^T = \mathbf{c} \]

\[ \text{where } \mathbf{c} \text{ and } \theta(t) \text{ are the initial state and initial state estimate, respectively.} \]

\[ \text{The choice of the filter } \mathbf{F}(t) \text{ is the optimal filter in the case of a colored measurement noise:} \]

\[ \text{where } k(t) \text{ is the least-square estimate of } x(t) \]

\[ \text{The proposed solution is to use the accurate model of the system, then find the controller of the dynamic.} \]

\[ \text{This model has two main characteristics:} \]

\[ \text{First - the filter part and the gain part of the control controller,} \]

\[ \text{Second - we can easily design the controller.} \]

\[ \text{Assume the following form of the controller:} \]

\[ \text{where } \mathbf{z}(t) \text{ is the least-square estimate of } x(t) \]

\[ \text{This problem has also a well-known solution using Kalman optimal filter:} \]

\[ \{ \phi(t) \theta(t) \}^T = \mathbf{c} \]

\[ \text{where } \mathbf{c} \text{ and } \theta(t) \text{ are the initial state and initial state estimate, respectively.} \]
The 7th National Conference of Electrical & Electronic Engineers in Israel

The solution of the following equation:

\[ y(t) = (e(t) - \sum_{j=1}^{n} a_j x(t-j)) e(t) \]

satisfies the following equation:

\[ y(t) = y(t-1) + y(t-2) + \ldots + y(t-\tau) \]

In order to solve the optimal control problem, it is worthwhile to transform the stochastic system to the following form:

\[ \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 & \cdots & A_n \end{bmatrix} \begin{bmatrix} x(t) \\ \vdots \\ x(t-n+1) \end{bmatrix} + \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} u(t) \]

The matrix \( w(t) \) can be decomposed in the following way:

\[ \begin{bmatrix} w(t) \\ \vdots \\ w(t-n+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 & \cdots & A_n \end{bmatrix} \begin{bmatrix} x(t) \\ \vdots \\ x(t-n+1) \end{bmatrix} + \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} u(t) \]

where

\[ A = \begin{bmatrix} A_1 \\ \vdots \\ A_n \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} \]
The 7th National Convention of Electrical & Electronic Engineers in Israel

suggested to the solution of the whole problem, and a digital computer program was developed. The solution, because of the problem involving a matrix and a system of linear equations, is not always exact and may be approximated by a numerical approach. The numerical method is based on the idea of approximating the solution by a sequence of linear equations, which are solved iteratively. The method is iterative, and the solution is improved by solving a sequence of linear equations. The solution is obtained by using the method of least squares, which minimizes the sum of the squares of the residuals.

\[
\begin{align*}
\mathbf{Z} & = \begin{bmatrix} \mathbf{L} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{u} & \mathbf{c} \end{bmatrix} \mathbf{X} = \mathbf{X} \\
\mathbf{Z}_0 & = \mathbf{d} \\
\mathbf{Z}_1 & = \mathbf{u} \\
\mathbf{Z}_2 & = \mathbf{d} \\
\mathbf{Z}_3 & = \mathbf{u}
\end{align*}
\]

Hence

\[
\begin{align*}
\mathbf{Z}_0 & = \mathbf{d} \\
\mathbf{Z}_1 & = \mathbf{u} \\
\mathbf{Z}_2 & = \mathbf{d} \\
\mathbf{Z}_3 & = \mathbf{u}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} \mathbf{Z}_0 & \mathbf{d} \mathbf{u} & \mathbf{Z}_1 & \mathbf{d} \mathbf{u} \\
\mathbf{X} & \mathbf{d} & \mathbf{c} \end{bmatrix} & = \begin{bmatrix} \mathbf{Z}_0 & \mathbf{d} \mathbf{u} & \mathbf{Z}_1 & \mathbf{d} \mathbf{u} \\
\end{bmatrix} = \mathbf{T}
\end{align*}
\]

Hence:

\[
\begin{align*}
\mathbf{Z}_0 & = \mathbf{d} \\
\mathbf{Z}_1 & = \mathbf{u} \\
\mathbf{Z}_2 & = \mathbf{d} \\
\mathbf{Z}_3 & = \mathbf{u}
\end{align*}
\]

This defines the optimal matrix \( \mathbf{T} \). In order to determine the matrices \( \mathbf{P} \) and \( \mathbf{K} \), we decom-

\[
\begin{align*}
\mathbf{Z}_0 & = \mathbf{d} \mathbf{u} \\
\mathbf{Z}_1 & = \mathbf{d} \mathbf{u} \\
\mathbf{Z}_2 & = \mathbf{d} \mathbf{u} \\
\mathbf{Z}_3 & = \mathbf{d} \mathbf{u}
\end{align*}
\]

where \( \mathbf{P} \) is a non-negative definite matrix satisfying the following matrix equation:

\[
\begin{align*}
\mathbf{P} & = \mathbf{Z}_0 \mathbf{d} \mathbf{u} \\
\mathbf{P} & = \mathbf{Z}_1 \mathbf{d} \mathbf{u} \\
\mathbf{P} & = \mathbf{Z}_2 \mathbf{d} \mathbf{u} \\
\mathbf{P} & = \mathbf{Z}_3 \mathbf{d} \mathbf{u}
\end{align*}
\]

Applying the algorithm of matrix multiplication and the least squares method, one can obtain the optimal solution. The solution can be expressed in the form:

\[
\begin{align*}
\mathbf{X} & = \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 \end{bmatrix} \\
\mathbf{Z}_0 \mathbf{d} & = \mathbf{X}_1 \\
\mathbf{Z}_1 \mathbf{d} & = \mathbf{X}_1 \\
\mathbf{Z}_2 \mathbf{d} & = \mathbf{X}_1 \\
\mathbf{Z}_3 \mathbf{d} & = \mathbf{X}_1
\end{align*}
\]

Similar expressions are obtained for the measurement noise, where it can easily be proved that

\[
\begin{align*}
\begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 \end{bmatrix} & = \mathbf{X}_0 \\
\mathbf{X}_1 & = \mathbf{X}_0 \\
\mathbf{X}_2 & = \mathbf{X}_0
\end{align*}
\]

where

\[
\begin{align*}
\mathbf{X}_0 & = \begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 \end{bmatrix} \\
\mathbf{X}_1 & = \begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 \end{bmatrix} \\
\mathbf{X}_2 & = \begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\mathbf{X}_0 & = \begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 \end{bmatrix} \\
\mathbf{X}_1 & = \begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 \end{bmatrix} \\
\mathbf{X}_2 & = \begin{bmatrix} \mathbf{X}_0 & \mathbf{X}_1 \end{bmatrix}
\end{align*}
\]

where \( \mathbf{A} \) is the covariance matrix of \( \mathbf{X} \). In this case the cost function has the form:

\[
\begin{align*}
\mathbf{C} & = \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_1 \\
\mathbf{C}_1 & \mathbf{C}_2 \\
\end{bmatrix} \\
\mathbf{C}_0 & = \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_1 \\
\mathbf{C}_1 & \mathbf{C}_2 \\
\end{bmatrix} \\
\mathbf{C}_2 & = \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_1 \\
\mathbf{C}_1 & \mathbf{C}_2 \\
\end{bmatrix}
\end{align*}
\]
It computes the matrices $P$, $K$, and $H$ of the optimal steady-state controller and the cor-

The program was written in Fortran IV language and later the conference student ent-

p-