The signal reliability of a system depends upon the nature of the possible faults and their probabilities of occurrence. We assume that the possible faults are multiple.

2. Preliminaries

Computationally feasible existence of such a steady state makes the analysis of sequential system reliability computationally infeasible. Since it implies reconstruction of the sequential system, this approach is computationally infeasible. The methods presented in [1–4] are restricted to combinatorial systems and the algorithm in [3], which can be applied to combinatorial and sequential systems, has been discussed [2–6]. However, the methods presented in [2] and [4] are restricted to polynomial applications of signal reliability have been given there. Recently, algorithms for the evaluation of this measure have been presented [2–4] and the applications of signal reliability have been given in [1–4].

Signal reliability as a measure of digital system's reliability was introduced by

1. Introduction

The model of a Markov chain, the reliability of a sequential system, the reliability of finite Markov chains, the reliability of the system's reliability, and the reliability of the system's reliability.

2. Abstract

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ITERATIVE AND SEQUENTIAL SYSTEMS
ON THE COVERAGE OF THE SIGNAL RELIABILITY OF

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fault. For permanent faults the most commonly used fault probability function is $S_x(t) = 1 - \exp(-\lambda_x t)$ where $\lambda_x$ is the failure rate at line $X$. For intermittent faults we select the continuous-parameter Markov model [7] shown in Figure 1. Here $S_x(t)$ is the probability that the intermittent fault is active at time $t$ and is given by

$$S_x(t) = S_x(0) \exp\left[-(\lambda_x + \mu_x)t\right] + \frac{\lambda_x}{\lambda_x + \mu_x} \left[1 - \exp\left[-(\lambda_x + \mu_x)t\right]\right]$$

(2.1)

where $\lambda_x(\mu_x)$ is the transition rate from the fault inactive (active) state to the fault active (inactive) state of the intermittent fault at line $X$.

The reliability of the system is clearly time-dependent. However, to simplify notation, we shall omit $t$ as an argument of the reliability and failure probability functions, and these functions are understood to be time-dependent.

The presence of faults in the system may cause incorrect logic signals on some lines. Consequently, the signal on any line $X$ in the system may assume one out of four values, namely: correct 0, correct 1, incorrect 0, and incorrect 1. These values are designated by 0, 1, 2 and 3, respectively. Thus, the signal on line $X$ is a four-valued logic signal and the probabilities of its four possible values are

$$\Pr\{X = 0\} = \Pr\{X \text{ is correctly a 0}\} \triangleq R_0(X)$$
$$\Pr\{X = 1\} = \Pr\{X \text{ is correctly a 1}\} \triangleq R_1(X)$$
$$\Pr\{X = 2\} = \Pr\{X \text{ is incorrectly a 0}\} \triangleq R_2(X)$$
$$\Pr\{X = 3\} = \Pr\{X \text{ is incorrectly a 1}\} \triangleq R_3(X)$$

These four probabilities form a reliability vector $R(X) = [R_0(X), R_1(X), R_2(X), R_3(X)]$ whose elements satisfy

$$R_0(X) + R_1(X) + R_2(X) + R_3(X) = 1.$$

The signal reliability of line $X$, denoted $SR(X)$, is the probability that the signal on line $X$ is correct, hence,

$$SR(X) = R_0(X) + R_1(X)$$

The signal reliability of a system, whose output is $Z$, is $SR(Z)$. This reliability is calculated from the input lines' reliability vectors using a reliability transfer function whose evaluation is based on the reliability model devised in [3]. In this model, the occurrence of faults is introduced through special elements called fault occurrence networks (FON). Such an element is inserted into each line of the system. Faults may occur in these elements only, and the rest of the system is considered fault-free.
calculated from the RMs of its components.

RTMs of some basic elements and then we show how the RMT of a system is calculated are the basic elements of the model. In the following we first derive the

smallest RTMs in an appropriate form. The RMT of a system is calculated using an RMT or a given system is decomposed into subsystems and an RMT

for multiple-column systems can be derived in a similar way.

The definition of an RMT is not restricted to single-output systems and an RMT

undecoupled matrix $J$.

The property we omit the justification for the row and column reduction and we use the

example to illustrate the justification for the row and column reduction.

For the sake of convenience we define $\mathbf{0}$, 0, $\mathbf{1}$, 1, $\mathbf{2}$, 2, $\mathbf{n}$, 1, $\mathbf{n}$, n, $\mathbf{n}^t$.

The reduced matrix contains only the first two columns of $J$ and the reduced matrix $\mathbf{0}$, 0, 0, 0, $\mathbf{1}$, 1, $\mathbf{2}$, 2, $\mathbf{n}$, 1, $\mathbf{n}$, n, $\mathbf{n}^t$.

shown that only a reduced matrix of size $\mathbf{2}^+\mathbf{n}$ is actually needed. The

size of the RMT increases rapidly with $\mathbf{n}$ the number of inputs. Nonetheless, it can be

size.

The matrix $J$ is called the reliability transfer matrix, abbreviated RMT.

The conditional probabilities (2.2)

\[ \begin{align*}
J \cdot (X)^A &= (Z)^R \\
(2.2)
\end{align*} \]

matrix $J$ of order $\mathbf{n}^t$, \( \mathbf{1} \times \mathbf{n}^t \). The rows of $J$ are the bases of a binary vector $\mathbf{1}$ and the columns of $\mathbf{0}$. For the sake of convenience we define $\mathbf{0}$, 0, $\mathbf{1}$, 1, $\mathbf{2}$, 2, $\mathbf{n}$, 1, $\mathbf{n}$, n, $\mathbf{n}^t$.

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The conditional probabilities (2.2)

\[ \begin{align*}
(\mathcal{H}) \cdot \mathcal{X} = (Z)^R \\
(2.2)
\end{align*} \]

value interchangeably. Hence,

vector $J$.

The sum is over all $\mathbf{n}^t$-column vectors of length $\mathbf{n}$.

\[ \begin{align*}
\mathcal{X}_1 \cdot \mathcal{X}_2 \cdot \mathcal{X}_3 \cdot \mathcal{X}_4 = (Z)^R \\
(2.2)
\end{align*} \]

vector assumed by $\mathbf{X}$. Each element of row of $\mathbf{n}^t$ can be expressed as follows

\[ \begin{align*}
= (Z)^R \\
(2.2)
\end{align*} \]

To simplify notation, we denote $\mathcal{X}$ by $\mathbf{X}$ and let $\mathcal{H}$ be denoted by $\mathcal{H}$.

The sum is over all $\mathbf{n}^t$-column vectors of length $\mathbf{n}$.

\[ \begin{align*}
= (Z)^R \\
(2.2)
\end{align*} \]

For a given system $J$ we calculate a reliability transfer function in a form of a

Signal Reliability of Iterative and Sequential Systems
FON

Let $X, Z$ be the input and output lines of a FON, respectively. The elements of the RTM $T_{\text{FON}}$ depend upon the type of faults assumed to occur at line $X$. If the possible faults are stuck-at-zero (s-a-0) and stuck-at-1 (s-a-1) with probabilities $q_{0x}$ and $q_{1x}$, respectively, satisfying $S_x = q_{0x} + q_{1x}$, then the elements of the RTM are

\[
\begin{align*}
t_{00} &= \Pr(Z = 0 \mid X = 0) = \Pr(Z \text{ is correctly a 0} \mid X \text{ is correctly a 0}) \\
&= \Pr(\text{No s-a-1 fault occurred}) = 1 - q_{1x} \\
t_{01} &= \Pr(Z = 1 \mid X = 0) = \Pr(Z \text{ is correctly a 1} \mid X \text{ is correctly a 0}) = 0 \\
\end{align*}
\]

In a similar manner, $t_{02} = 0$.

\[
\begin{align*}
t_{03} &= \Pr(Z = 3 \mid X = 0) = \Pr(Z \text{ is incorrectly a 1} \mid X \text{ is correctly a 0}) \\
&= \Pr(\text{A s-a-1 fault occurred}) = q_{1x} \\
\end{align*}
\]

Similarly, all other elements of $T_{\text{FON}}$ are calculated, yielding

\[
T_{\text{FON}} = \begin{bmatrix}
1 - q_{1x} & 0 & 0 & q_{1x} \\
0 & 1 - q_{0x} & q_{0x} & 0 \\
0 & q_{1x} & 1 - q_{1x} & 0 \\
q_{0x} & 0 & 0 & 1 - q_{0x}
\end{bmatrix}
\quad (2.5)
\]

If the possible fault is an "inverted signal" fault (i.e., $Z = X'$) with probability $S_x$, the resulting RTM is

\[
T_{\text{FON}} = \begin{bmatrix}
1 - S_x & S_x & 0 & 0 \\
S_x & 1 - S_x & 0 & 0 \\
0 & 0 & 1 - S_x & S_x \\
0 & 0 & S_x & 1 - S_x
\end{bmatrix}
\]

In both cases the lead failures are not necessarily permanent.

Similarly, a matrix $T_{\text{FON}}$ can be derived for any other kind of lead failure. Furthermore, the type of fault and its probability have not necessarily to be the same for the various leads in the system. Some of the leads may even be fault-free wires (i.e., $S_x = 0$) yielding

\[
T_{\text{FON}} = I
\]

NOT gate

Let $X, Z$ be the input and output lines of a fault-free NOT gate. Then,

\[
\begin{align*}
t_{00} &= \Pr(Z = 0 \mid X = 0) = \Pr(Z \text{ is correctly a 0} \mid X \text{ is correctly a 0}) = 0 \\
t_{01} &= \Pr(Z = 1 \mid X = 0) = \Pr(Z \text{ is correctly a 1} \mid X \text{ is correctly a 0}) = 1 \\
t_{02} &= \Pr(Z = 2 \mid X = 0) = \Pr(Z \text{ is incorrectly a 0} \mid X \text{ is correctly a 0}) = 0 \\
t_{03} &= \Pr(Z = 3 \mid X = 0) = \Pr(Z \text{ is incorrectly a 1} \mid X \text{ is correctly a 0}) = 0
\end{align*}
\]
Theorem 2.1 can be generalized to the case where \( J \) subsystems \( j \) exist and is defined as

\[
\mathcal{O} V \cdot \mathcal{O} (V \otimes (V \mathcal{O})) = J
\]

where the star product \( \star \) is a generalization of the Kronecker matrix product and is

\[ (V \mathcal{O}) \star (V \mathcal{O} \otimes (V \mathcal{O})) = J \]

Theorem 2.1: Let \( J \) be the RIM of the system in Figure 2, respectively. The RIM of the system is

\[ J = \mathcal{O} \text{ when } \mathcal{O} \text{ is a fault-free system} \]

as is a 4 \times 4 matrix

Thus, the elements of \( \mathcal{O} \) are

\[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

Thus,

\[ I = \{ X = \{ Z \} \} \mathcal{O} = \{ \varepsilon \} \mathcal{O} \]

Besides the other non-zero elements of \( \mathcal{O} \) are

\[ I = \{ I = \{ Z \} \} \mathcal{O} = \{ \varepsilon \} \mathcal{O} \]

In a similar way, we may calculate the other elements of \( \mathcal{O} \).

The RIM of any other fault-free state can be computed in a similar fashion.
Theorem 2.2 [4]: The RTM of a system constructed of $l$ subsystems feeding $M_0$ is

$$T = (A^{(1)} \otimes A^{(2)} \otimes \ldots \otimes A^{(l)}) \cdot A^{(0)}$$

It is shown in [4] that theorem 2.2 can be applied to an arbitrary system after proper partitioning of the system into subsystems. This partitioning is applied recursively until a level is reached where the RTMs of the subsystems are known.

This method of calculating the RTM of a given system using the RTMs of its subsystems is especially attractive in the following cases:

1. The system is constructed of standard LSI modules. In this case, if a standard module is used more than once throughout the system, its RTM has to be calculated just once.
2. The system consists of several identical subsystems, e.g. cellular arrays and NMR systems.

The second case is illustrated in the following example.

Consider a TMR configuration of a Full-Adder. The RTM of the TMR system, denoted by $T_{TMR}$, is calculated using the RTM of a single Full-Adder, $T_{FA}$, and the RTM of the Majority-Voter, $T_V$, as follows

$$T_{TMR} = (T_{FA} \otimes T_{FA} \otimes T_{FA}) \cdot T_V$$

The extension of this equation to NMR system is straightforward.

In the next section we consider iterative arrays. The final results obtained in that section are extended later to the synchronous sequential systems in section 4.

3. Iterative systems

In this section we investigate the signal reliability of a cell in an iterative system and the conditions under which this reliability converges toward a steady state value. When such a convergence takes place a considerable reduction in the amount of computation needed is achieved.

Let $X = X_1, X_2, \ldots, X_n$, $Y = Y_1, Y_2, \ldots, Y_m$ and $Z = Z_1, Z_2, \ldots, Z_l$ be the vectors of $n$ primary input lines, $m$ carry lines between the cells and $l$ primary output lines, respectively, of a typical cell in an iterative system. Let $T_Y(T_Z)$ be the RTM of
The finite Markov chain $Q$ of a Full-adder.

Figure 4: The state diagram $D$ of a Full-adder.

Figure 3: The state diagram $D$ of a Full-adder.

Since the primary inputs are independent of the carry signals, the second probability in (2.3) equals $P(X)$. Hence, the first probability in (2.3) is, by definition, the element $P(X_{1}=0|X_{0}=0)$. Note that the carry (out) is an example of the expansion of $D$ in the transition probability $P$. We rewrite the transition probability as follows:

$$P(Y_{1}=dX_{1} | Y_{0}=X_{0})P_{d} = P(Y_{1}=dX_{1} | Y_{0}=X_{0}, \bar{X}_{0}=X_{0}, \bar{Y}_{0}=Y_{0})P_{d} = P(Y_{1}=dX_{1} | Y_{0}=X_{0})P_{d}$$

In the cell, an edge from $i$ to $j$ in $S_Q$ is labeled with the transition probability $\alpha_{ij}$ in $Q$. In $S_{Q}$, the additional transitions are due to faults present in $S_{Q}$ and faults in $Q$. For example, the expanded $\alpha_{ij}$ is labeled with $\alpha_{ij}$ and $\alpha_{ij}$ are due to faults present in $S_{Q}$ and faults in $Q$, respectively. Further, the expanded $\alpha_{ij}$ corresponds to $i = N_i$ and $j = Y_{1}$, where $Y_{1}$ is the state corresponding to $i = N_i$. When faults are introduced into the cell, we replace the two-valued signals by four-valued signals. The carry output (primary output) of the cell, the logical operation of the fault-free cell can be described by a deterministic state diagram with $2^4$ states. $S_{Q}$ can be described by a deterministic state diagram with $2^4$ states.
$t(i, k), j$ of the RTM $T_Y$ where $(i, k)$ is the concatenation of $i$, the present state vector, and $k$, the primary input vector.

Hence,

$$P_{ij} = \sum_{k=0}^{4^n-1} t(i, k), j \Pr(X = k)$$  \hspace{1cm} (3.3)

Thus, if the primary input sequence has a stationary (time invariant) probability then $D_U$ is a probabilistic state diagram that describes a finite Markov chain. The properties of this Markov chain depend upon the transition matrix $P = \{P_{ij}\}$ which from (3.3) depends on the RTM $T_Y$ which in turn depends upon the possible faults in the cell and their probabilities of occurrence. To investigate the properties of the Markov chain we define the following function

$$\phi(X) = \begin{cases} 
0 & \text{if } X = 0 \text{ or } 3 \\
1 & \text{if } X = 1 \text{ or } 2
\end{cases}$$

i.e. $\phi(X)$ is the correct value of $X$. We extend the definition of $\phi$ to vectors $X$ by applying $\phi$ to each element separately. When applied to the states of $D_U$ the function $\phi$ defines a relation $E$ as follows, two states $U_i$ and $U_j$ satisfy

$$(U_i, U_j) \in E \quad \text{iff} \quad \phi(U_i) = \phi(U_j)$$  \hspace{1cm} (3.4)

The relation $E$ is clearly an equivalence relation and each equivalence class contains $2^n$ states, one of which is a correct state designated $U_{i*}$ satisfying $U_{i*} = \phi(U_{i*})$.

The function $\phi$ is employed in the proof of the following lemma showing that the Markov chain $D_U$ is irreducible if $D_S$ is strongly connected and the failures in all the input and output carry leads have non-zero probabilities. If some of these faults have zero probabilities, it is possible to obtain a reducible Markov chain $D_U$ even if $D_S$ is strongly connected. However, the above condition is satisfied in most practical cases.

**Lemma 3.1:** If the faults in all the input and output carry leads have non-zero probabilities then the finite Markov chain $D_U$ is irreducible iff the cell state diagram $D_S$ is strongly connected.

**Proof:** A finite Markov chain is irreducible, i.e. its state diagram is strongly connected, if for any two states $U_i$ and $U_j$ there is a directed path from $U_i$ to $U_j$. We first prove that there is an edge from $U_i$ to $U_j$ in $D_U$ iff there is an edge from $U_{i*}$ to $U_{j*}$. To prove the only if part suppose that an edge from $U_i$ to $U_j$ exists, meaning that a cell with an input carry $U_i$ may produce an output carry $U_j$. If no faults exist then the input carry to this cell is $U_{i*}$ and the resulting output is some correct state $U_k$. In the presence of faults the actual output carry may be either $U_k$ or some incorrect output carry which is $\phi$-equivalent to $U_k$ (under relation $E$). Since $U_j$ is such an incorrect output carry we have $\phi(U_j) = \phi(U_{k*}) = U_{j*}$. To prove the if part consider a cell with a correct input carry $U_{i*}$ and a correct output carry
The signal reliability of the primary input $X$ is assumed.

A similar construction is achieved by the primary output's signal reliability $S(X)_Z$ since a stationary probability of the point $X$ is steady-state value.

Iterative system convergence to a steady-state value is the above condition, the signal reliability $S(Y)_S$ of the output carry of a cell in an iterative system is correct, therefore, if the sum of the correct state's probabilities, hence, under correction, the correct state is the output carry of the cell in the iterative system is

$$S(Y)_S = \sum_{D \in \Omega} p(D) = \sum_{D \in \Omega} \frac{1}{1 - \rho(D)}$$

and

$$S(Y)_S = \sum_{D \in \Omega} \frac{1}{1 - \rho(D)}$$

The signal reliability of the output carry is the probability that the output carry is correct.

Proof: Immediate from Lemma 3.2.

Corollary: $S(Y)_S$ is periodic if $D$ is periodic.

Lemmas

Lemma 3.2: The periodicity of $D$ is equivalent to $S(Y)_S$.

Proof: The periodicity of $D$ is equivalent to $S(Y)_S$.

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Proof: The periodicity of $D$ is equivalent to $S(Y)_S$.

Signal reliability of iterative and sequential systems.
probabilities are given by equation (3.3). The primary input signals that appear in (3.3) are assumed to be either correct 0 or correct 1 with equal probabilities, i.e. \( R(X) = (0.5, 0.5, 0, 0) \). The numerical results have shown that convergence of the signal reliability is achieved within 13 stages, i.e. stages 13–40 have the same signal reliabilities, namely, output carry reliability of \( SR(Y) = 0.93841 \) and primary output reliability of \( SR(Z) = 0.82374 \). The main conclusions drawn from this example are that the signal reliability of a parallel adder with a reasonable number of stages is independent of the number of stages and that a larger number of stages does not imply a lower reliability. The latter conclusion is in contrast to the conclusion drawn when the functional reliability measure is employed. When evaluating the functional reliability of a system, the reliability of the basic cell of the system (a Full-Adder stage in this case) is raised to the power of \( k \) where \( k \) is the number of these cells in the system. Consequently, the functional reliability of a parallel adder is a decreasing function of the number of stages.

4. Synchronous sequential systems

To analyse the signal reliability of synchronous sequential systems we model them as infinite iterative systems. This enables us to extend the results of the previous section to apply to synchronous systems. The modelling is illustrated in Figure 5. Part (a) shows a sequential system with a single JK flip-flop. Part (b) shows a typical cell of the iterative system model. The flip-flop equivalent in (b) is a combinational circuit whose RTM is identical to the RTM \( T_{JKFF} \) (developed in section 2). It calculates the next state \( Y_N \) from the present state \( Y_P \) and the excitation signals \( J \) and \( K \).

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Fig. 5. Modelling a synchronous sequential system by an iterative system.
Within 10 clock cycles, yielding an output signal reliability of SR(2) = 0.9812 and a modelled by an iterative system. The results show a slight decrease in the standby power due to increased standby power consumption and endurance. A simple test is run to calculate the signal reliability of the serial adder circuit, assuming that the condition is strongly connected and periodic (see Figure 3). This sequential system is clearly significant, since the probabilities of the initial outputs are just 0.1 and the probabilities that a single fault on multiple intermittent faults all fit into its bounds. The probability that a single flip-flop is subject to occur...

**Example:** A serial adder containing a single YF flip-flop is subject to a failure of 0.1. In the following example, we illustrate the reliability of the signal reliability of a sequential system.

On the other hand, it is a matter of a few clock cycles. The reliability of the state probabilities on the fault location varies a weak function of time and it takes weeks or months of on-line fault injection to reveal the potential of a fault. The fault probability is not the fault probability for in the fault probability given by (1). The fault probability is strongly connected, periodic, and all its transitions are valid. The fault probability is strongly connected, periodic, and all its transitions are valid.

For every line x in the cell where 0.8 < x < 1, the fault frequency...

**Condition:** For every fault x, the condition is satisfied if and only if the Markov model is exponentially distributed with mean and variance 1/\( \lambda \). Hence, this condition is satisfied if...

Parameter Markov model is exponentially distributed with mean and variance 1/\( \lambda \). The duration of an intermittent fault characterized by the continuous-state Markov chain converges to steady-state values if the state transitions of the intermittent faults satisfy the condition (1).
carry signal reliability of $SR(Y) = 0.9620$. The values of the signal reliability in the first 10 clock cycles depend upon the reliability of the flip-flop's initial state. These values have been calculated first for $R(Y_0) = (1, 0, 0, 0)$ (i.e. the flip-flop is in the reset state with probability 1) and then for $R(Y_0) = (0.5, 0, 0, 0.5)$. The results are summarized in Figure 6 illustrating the speed of convergence.

5. Conclusions

The signal reliabilities of iterative and sequential systems have been analysed in this paper. It has been shown that, under certain conditions, the signal reliabilities of these systems converge to steady state values, thus avoiding the need for prohibitively large number of calculations. Furthermore, this convergence provides a new insight when the signal reliability measure is compared to the more pessimistic functional reliability measure as illustrated in the example in section 3.

References


La méthode du signal est une mesure précise de la qualité des systèmes de communication.