UNIVERSITY OF MASSACHUSETTS
Department of Electrical and Computer Engineering

ECE 655 Fault Tolerant Computing

Sample Mid-term Exam

Open lecture notes and graded homework assignments **only**.
Solve all four problems. You have 2 hours.
Write your answers in the provided space.
Show your work and clearly mark your final answer.

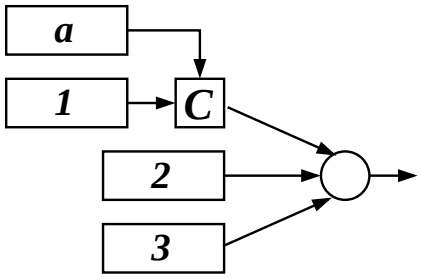
1 _____ (20 points)
2 _____ (30 points)
3 _____ (20 points)
4 _____ (30 points)
Total _____ (100 points)

Student name : _____

1. (a) (8 points) A certain cyclic code is based on the generator polynomial X^4+X+1. What will be the code word for the data bits $a_5a_4a_3a_2a_1a_0 = 101101$?

(b) (12 points) Four bit errors have occurred in the code word in (a) resulting 1101000111. Show the decoding process. Were the errors detected? Will all multiple bit errors of up to four be detected? List the errors which are guaranteed to be detected.

2. (a) (15 points) The system shown consists of a TMR core with a single spare a which can serve as a spare only for module 1. Assume that modules 1 and a are active. When either of the two modules 1 or a fails, the failure is detected by the perfect comparator C and then the single operational module is used to provide an input to the voter. Assuming that the voter is perfect as well, which one of the following expressions for the system reliability is correct (where each module has a reliability R and the modules are independent). Explain your answer. A correct answer with either no explanation or an incorrect one is worth only 2 points.



(1) $R_{system} = R^4 + 4R^3(1-R) + 3R^2(1-R)^2$
(2) $R_{system} = R^4 + 4R^3(1-R) + 4R^2(1-R)^2$
(3) $R_{system} = R^4 + 4R^3(1-R) + 5R^2(1-R)^2$
(4) $R_{system} = R^4 + 4R^3(1-R) + 6R^2(1-R)^2$

(b) (15 points) Write an expression for the reliability of the system if instead of a perfect comparator for modules 1 and a there is a coverage factor c, i.e., c is the probability that a failure in one module is detected, the faulty module is correctly identified and the operational module is successfully connected to the voter (which is still perfect).

3. (a) (10 points) A certain $(7, 4)$ SEC Hamming code is defined by the following parity check matrix:
Write the Boolean equations for the three parity bits and

	p_2	p_1	p_0	a_3	a_2	a_1	a_0
p_2	1	1	1	0	0	0	1
p_1	1	1	0	1	0	1	0
p_0	1	0	1	0	1	1	0

express the parity bits (p_2, p_1, p_0) as functions of the data bits (a_3, a_2, a_1, a_0) only.

(b) (5 points) Show the encoding of the data bits $a_3, a_2, a_1, a_0 = 0101$.

(c) (5 points) The encoded word $(p_2, p_1, p_0, a_3, a_2, a_1, a_0 = 0011001$ has a single bit-error. What is the correct codeword?

4. (a) (15 points) A duplex system consists of two active units and a comparator. Assume that each unit has a failure rate of λ and a repair rate of μ. The outputs of the two active units are compared and when a mismatch is detected, a procedure to locate the faulty unit is performed. The probability that upon a failure, the faulty unit is correctly identified and the fault-free unit (and consequently, the system) continues to run properly, is denoted by c and is called the coverage factor. Note that when a coverage failure occurs, the entire system fails and then both units have to be repaired (at a rate μ each). When one unit is repaired, the system become operational and the repair of the second unit continues allowing the system to return to its original state.
Show the Markov model for the duplex system.

(b) (15 points) Derive an expression for the steady-state availability of the system assuming that $\mu = 2\lambda$.