FAULT TOLERANT SYSTEMS

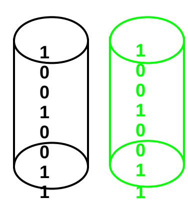
http://www.ecs.umass.edu/ece/koren/FaultTolerantSystems

Part 8 – RAID Systems

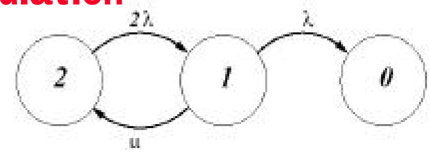
Chapter 3 – Information Redundancy

RAID - Redundant Arrays of Inexpensive (Independent) Disks

- **RAID1** - two mirrored disks
- If one disk fails, the other can continue
- If both work:
  - speeds up read accesses - divides them among two disks
  - Write accesses are slowed down
- Computing reliability, availability, and MTTDL (mean time to data loss) of **RAID1**



RAID1 - Reliability Calculation



- **Assumptions:**
  - * disks fail independently
  - * failure process - Poisson process with rate l
  - * repair time - exponential with mean time 1/m
- **Markov chain:** state - number of good disks

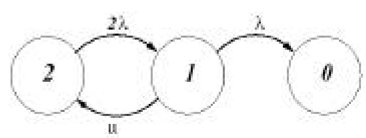
$$\frac{dP_2(t)}{dt} = -2l\,P_2(t) + mP_1(t) \qquad \frac{dP_1(t)}{dt} = -(l+m)P_1(t) + 2l\,P_2(t)$$

$$P_0(t) = 1 - P_1(t) - P_2(t) \qquad P_2(0) = 1; \qquad P_0(0) = P_1(0) = 0$$

- **Reliability at time t** -

$$R(t) = P_1(t) + P_2(t) = 1 - P_0(t)$$

RAID1 - MTTDL Calculation

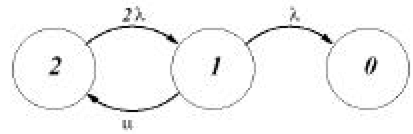


- Starting in state 2 at t=0
  - mean time before entering state 1 = 1/(2l)
- Mean time spent in state 1 is 1/(l + m)
- Go back to state 2 with probability q = m /(m + l)
  or to state 0 with probability p = l /(m + l)
- Probability of n visits
  to state 1 before transition to state 0 is $q^{n-1}p$
- Mean time to enter state 0 :

$$T_{2\to 0}(n) = n\left(\frac{1}{2l} + \frac{1}{l+m}\right) = n\frac{3l+m}{2l(l+m)}$$

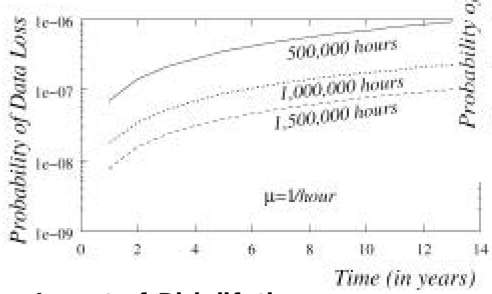
$$MTTDL = \sum_{n=1}^{\infty} q^{n-1} p T_{2\to 0}(n) = \sum_{n=1}^{\infty} n q^{n-1} p T_{2\to 0}(1) = \frac{T_{2\to 0}(1)}{p} = \frac{3l+m}{2l^2}$$

Approximate Reliability of RAID1

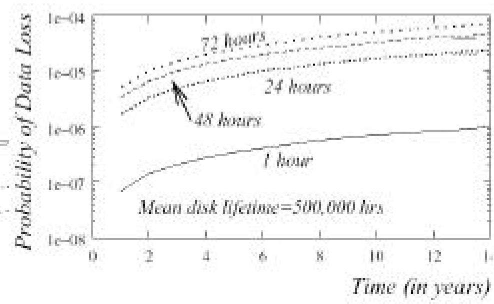


- If m >> l , the transition rate into state **0** from the aggregate of states **1** and **2** is **1/MTTDL**
- Approximate reliability:

$$R(t) = e^{-t/MTTDL}$$

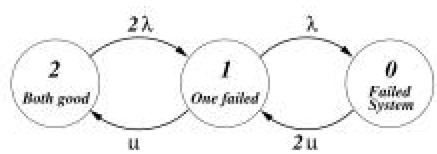


Impact of Disk lifetime



Impact of Disk Repair time

RAID1 - Availability Calculation



- **Markov chain:** state - number of good disks
- The solution to the differential equations:

$$P_2(t) = m^2/(l+m)^2 + 2lm/(l+m)^2 e^{-(l+m)t} + l^2/(l+m)^2 e^{-2(l+m)t}$$

$$P_1(t) = 2lm/(l+m)^2 + 2l(l-m)/(l+m)^2 e^{-(l+m)t} - 2l^2/(l+m)^2 e^{-2(l+m)t}$$

$$P_0(t) = 1 - P_2(t) - P_1(t)$$

- **Long-term availability** -

  A = P_2 + P_1 = 1 - P_0

$$= (m^2 + 2lm)/(l+m)^2 = 1 - l^2/(l+m)^2$$

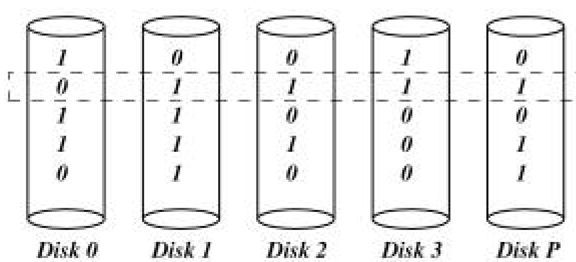
RAID2

- A bank of data disks in parallel with Hamming-coded disks
- d data disks and c code disks
- i-th bit of each disk - bit of a c+d-bit word
- From Hamming codes theory - to permit the correction of one bit per word -

$$2^c \geq c + d + 1$$

- We will not spend more time on RAID2 because other RAID designs impose much less overhead

RAID3



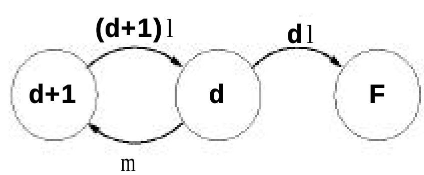
- Modification of RAID2
- Observation - each disk has error-detection coding per sector - a bad sector can be identified
- Bank of d data disks together with one parity disk
- Data are bit-interleaved across the data disks
- The i-th position of the parity disk contains the parity bit associated with the bits in the i-th position of each of the data disks

Error Detection and Correction in RAID3

- **i-th** bits of each disk form a **d+1**-bit word - **d** data and **1** parity bits
- If **j-th** bit in word is incorrect - sector error-detecting code in **j-th** disk will indicate a failure - fault will be located - remaining bits can be used to restore the faulty bit
- **Example**: word - **11100 ;** data bits - **1110** ; parity bit - **0**
- If even parity is being used - a bit is in error
- Third disk indicates an error in the relevant sector and the other disks show no such errors
- The correct word is **11000**

Reliability of RAID3

- Similar analysis to **RAID1**: **(d+1)** disks instead of **2**



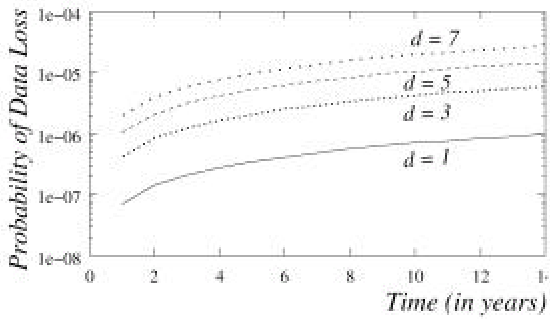
- System fails (data loss) if **two** or more disks fail
- Mean time to data loss for this group is

$$MTTDL = \frac{(2d+1)\,l + m}{d\,(d+1)\,l^2}$$

- The reliability is given approximately by

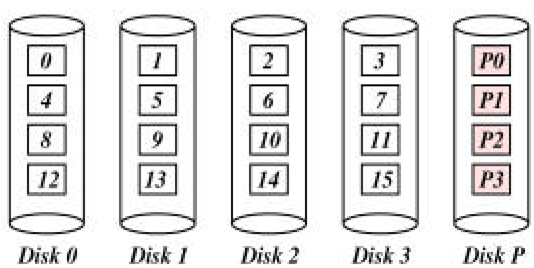
$$R(t) = e^{-t/MTTDL}$$

Comparing Different RAID3 Systems



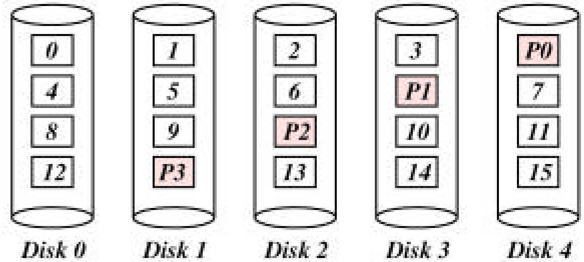
- Unreliability of RAID3 systems for different values of d - mean lifetime of a single disk is 500,000 hours
- The d=1 case - identical to RAID1
- Reliability goes down as d increases

RAID4



- Similar to RAID3 - but unit of interleaving block of arbitrary size - a stripe
- Advantage - a small read may be contained in one single data disk, rather than interleaved over all disks
- Small read operations are faster in RAID4
- Similarly for small write operations
- Write - affected data disk and parity disk must be updated
- Parity update simple - parity bit toggles if data bit being written is different from one being overwritten
- Reliability model for RAID4 - identical to RAID3

RAID5



- Observation - parity disk can be system bottleneck
- In RAID4 - parity disk accessed in each write
- In RAID5 - parity blocks interleaved among disks
- Every disk has some data blocks and some parity blocks
- Reliability model for RAID5 same as for RAID4
- Only the performance model is different

Modeling Correlated Failures

- We assumed until now that disks are independent with respect to failures
- Disk failures may be correlated - power supply and control are typically shared among multiple disks
- Disk systems are usually made up of strings - consisting of disks that share power supply, cabling, cooling, and a controller
- If any of these shared support items fail, the entire string can fail
- If the string constitutes the RAID group - data loss can occur

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Approximate Reliability of String

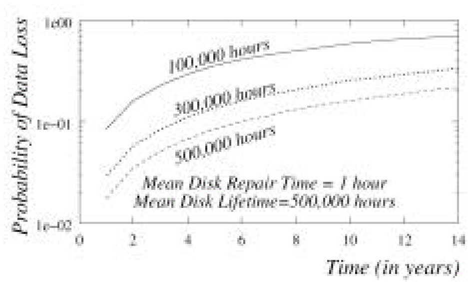
- l_{str} - failure rate of the support elements power, cabling, cooling, control) of a string
- l_{indep} - approximate failure rate due to independent failures
- If a RAID group is controlled by a single string - the aggregate failure rate of the group is

$$l_{total} = l_{indep} + l_{str}$$

- And the reliability is

$$R_{total}(t) = e^{-l_{total} t}$$

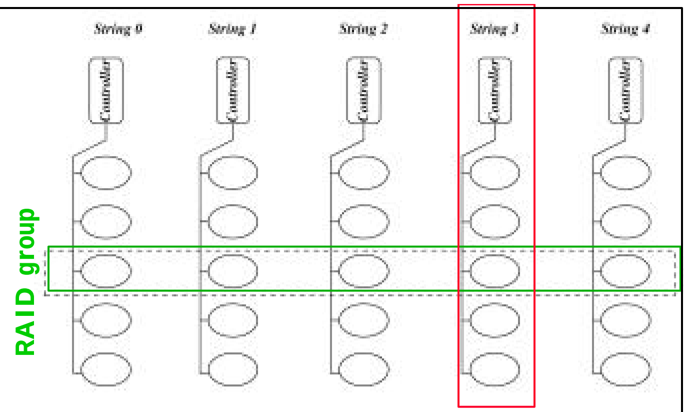
Impact of String Failures on RAID1



Mean String Lifetime

- Similar results for RAID3 and higher levels
- Figures of 150,000 hours for the mean string lifetime have been quoted in the literature
- At least one manufacturer claims mean disk lifetimes of 1,000,000 hours
- Grouping an entire RAID array as a single string increases unreliability by orders of magnitude

Orthogonal Arrangement of Strings and RAID Groups



- Failure of a string affects only one disk in each RAID group
- Since each RAID can tolerate the failure of up to one disk, this reduces the impact of string failures
- Data loss will happen only if any RAID group has at least two disks down at the same time

Part.8 .17

Approximate Modeling of Orthogonal Systems

- Data loss is caused by a sequence of events
- A failure can be triggered by an individual disk failure or by a string failure - very low failure rates
- We will find the (approximate) failure rate due to each - Λ_{indiv}; Λ_{str}
- Sum of these two failure rates - the approximate overall failure rate - Λ_{data_loss}

  $\Lambda_{data\_loss} = \Lambda_{indiv} + \Lambda_{str}$
- It can then be used to approximately determine MTTDL - Mean time to data loss, and reliability - probability of no data loss over any given period of time

Part.8 .18

Orthogonal Arrangement - Notations

- **d+1** strings, **g RAID** groups - total of **(d+1)g** disks
- $f_{disk}(t)$ - density function of the disk repair time
- l_{disk} - failure rate of a single disk
- p_{indiv} - probability that a given individual failure triggers data loss
- Approximate rate (per disk) at which individual failures trigger data loss – $l_{disk} p_{indiv}$
- p_{indiv} = probability that a second disk fails in the affected **RAID** group while the first failure is not yet repaired
- This second failure has the rate $d(l_{disk} + l_{str})$ - the second disk failure can happen either due to an individual disk or string failure

Calculating Failure Rates - Λ_{indiv}

- Conditioning on t - repair time of first disk failure

$$\Pr ob\{Data \ \ Loss \,|\, repair \ \ takes \ \ t\} = 1 - e^{-d(l_{disk} + l_{str})t}$$

- Unconditional probability of data loss -

$$\begin{aligned}\pi_{indiv} &= \int_0^\infty \text{Prob}\{\text{Data loss}|\text{the repair takes } \tau\} \cdot f_{disk}(\tau)\,d\tau \\ &= \int_0^\infty (1 - e^{-d(\lambda_{disk}+\lambda_{str})\tau}) f_{disk}(\tau)\,d\tau \\ &= \int_0^\infty f_{disk}(\tau)\,d\tau - \int_0^\infty e^{-d(\lambda_{disk}+\lambda_{str})\tau} f_{disk}(\tau)\,d\tau \\ &= 1 - F^*_{disk}(d[\lambda_{disk} + \lambda_{str}])\end{aligned}$$

- $F^*_{disk}(.)$ - the Laplace transform of $f_{disk}(.)$
- Approximate rate at which data loss is triggered by individual disk failure -

$$\Lambda_{indiv} \approx (d+1) g \lambda_{disk} \{1 - F^*_{disk}(d[\lambda_{disk} + \lambda_{str}])\}$$

Calculating Failure Rates - Λ_{str}

- **Total rate of string failures:** $(d+1)\, l_{str}$
- **When a string fails - we repair the string, and any individual disks affected by this string failure**
- **Pessimistic assumption - a second failure can happen at any group or disk before all groups are fully restored**
- **Example: arrival of a second string failure to the same string before first failure has gone**
- **Optimistic assumption: disks affected by string failure are immune to further failures before string and affected disks are fully restored**
- **The difference between failure rates determines how tight bounds are**

Pessimistic Calculation

- **t - (random) time taken to repair the failed string and all disks affected by it**
- $f_{str}()$ **- probability density function of t**
- $F^*_{str}()$ **- Laplace transform of** $f_{str}()$
- **Pessimistic assumption - rate of additional failures**

$$l_{pess} = (d+1)\, l_{str} + (d+1) g\, l_{disk}$$

- **Conditioning upon t - the probability of data loss**

$$p_{pess} = 1 - e^{-l_{pess} t}$$

- **Integrating on t - unconditional pessimistic probability of data loss**

$$p_{pess} = 1 - F^*_{str}(l_{pess})$$

Optimistic Calculation

- Optimistic assumption - rate of additional failures

$$l_{opt} = d\, l_{str} + dg\, l_{disk}$$

- Conditioning upon t - the probability of data loss is

$$p_{opt} = 1 - e^{-l_{opt} t}$$

- Integrating on t - unconditional optimistic probability of data loss

$$p_{opt} = 1 - F^{*}_{str}(l_{opt})$$

Reliability of Orthogonal System

- Rate of string failures triggering data loss –

$$\Lambda_{str} = (d+1)\, l_{str}\, p\,; \quad (p_{pess} \quad or \quad p_{opt})$$

- Approximate rate of data loss in the system -

$$\Lambda_{data\_loss} \approx \Lambda_{indiv} + \Lambda_{str}$$

- Mean Time To Data Loss -

$$MTTDL \approx \frac{1}{\Lambda_{data\_loss}}$$

- System reliability - $R(t) \approx e^{-\Lambda_{data\_loss} t}$